

Constraining the Galactic Dynamo

TurISM: Direct Simulations of the Turbulent Interstellar Medium

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Scientific context

1 Introduction

The last decade with its major advances in observational technology has brought increasingly detailed maps of galactic magnetic fields. Radio-polarization data reveals the existence of turbulent as well as large scale fields within the interstellar medium (ISM). The main drivers of the turbulence are thought to be winds of massive stars and explosions of supernovae (SNe) (MacLow & Klessen 2004), as well as (in less active regions) the magneto-rotational instability (MRI) (Dziourkevitch, Elstner & Rüdiger 2004; Piontek & Ostriker 2005).

The explanation of galactic magnetic field structures is of considerable interest, especially as galaxies (unlike stars) comprise a class of astrophysical objects with a directly observable internal flow structure. The linkage between kinematic and magnetic structures is the heart of dynamo theory which tries to explain the generation of ordered fields from the underlying turbulent flow.

Dynamo theory argues that observed fields are due to a dynamic process (contemporary field), as opposed to a frozen-in field stemming from the formation of the galaxy (primordial field). This is justified by the observed high values of turbulent diffusivity (approx. 10^{26} cm²s⁻¹) which would lead to a decay of any ordered magnetic field component within about 0.7Gyr (Rohde, Elstner & Rüdiger 1998). To overcome this diffusive process astrophysicists seek for an adequate source for the production of magnetic field, see e.g. Beck et al. (1996) for a review.

The generation of azimuthal fields is easily explained by differential rotation which is the dominating galactic flow pattern. Any radial field will be instantly sheared out into the azimuthal direction. Observations, however, indicate large pitch-angles for the magnetic field, i.e. the directions of the magnetic field lines deviate strongly (up to 35°) from the direction of the velocity field. To explain these large angles one also needs a robust mechanism to re-generate the radial field (Elstner 2005).

The generation of poloidal field from toroidal field is termed alpha-effect and can be modelled as a mean electromotive force (EMF). The origin of this EMF lies in correlations of small-scale turbulent velocity and magnetic field, i.e., only a flow that exhibits some form of broken symmetry can produce a non-vanishing EMF. In the case of the ISM there are three characteristic asymmetries: (i) the axis of rotation, (ii) the (inward) galactic shear gradient, and (iii) the (vertical) gradient in density and turbulence intensity (Rüdiger & Kichatinov 1993). The first contribution is due to the Coriolis-force that will give uprising turbulent eddies a definite skewness. This leads to a non-zero mean kinetic helicity which will, in turn, produce the desired poloidal magnetic field component (Rüdiger & Hollerbach 2004). The effect of shear has already been explained above. The role of the third term is the expulsion of the generated field into large galactic heights (diamagnetic pumping) counteracting its production. Thus it is crucial to determine the mutual strength of terms (i) and (iii) to answer the question whether a galactic dynamo may operate.

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2 Analytical models and their limitations

Until recently, a direct numerical simulation of the turbulent ISM has been prohibitive. Therefore early theoretical models (Rüdiger & Kichatinov 1993) tried to predict the outcome of supernova-driven turbulence based on simplifying assumptions. Ferrière (1992a) analytically derived the dynamo effect for isolated SN-explosions. The effects of explosions at various galactic heights z were then convolved with an assumed vertical SN distribution. This gave an alpha-effect with a magnitude of a few m/s while turbulent pumping (described by the escape velocity V_{esc}) dominated this effect by a factor of about 50, thus rendering dynamo action highly impossible.

Because single SNe turned out to be much too weak, also the effects of several correlated SNe, so-called superbubbles (SBs) were considered, though, still as single, non-interacting events. This increased the coherence time of the raising bubbles and thus the amplitude of the alpha-effect to about 400m/s (Ferrière 1998) but the pumping effect still remained dominant by a factor of 15.

The major limitation to all these models results from the fact, that the explosions are considered as non-interacting, isolated events. Collisions of expanding supernova remnants (SNR), however, lead to strong fragmentation and subsequently to the formation of over-dense filaments and clumps. These structures in turn will rain back into the gravitational potential (mainly of the stellar component) comprising the so-called galactic fountain. Because of the high conductivity of the ionized ISM the magnetic flux is frozen into the fluid and, thus, hindered from escaping the galaxy. This complex effect, that can only be captured in fully dynamic MHD simulations, might reduce the value of V_{esc} and thus make dynamo action possible at all.

References:

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Aspired scientific aims

The primary focus of this work is on the galactic dynamo and the generation of large-scale fields. To date, there exist quite elaborate models for the turbulent ISM, e.g., Joung & Mac Low (2006) or de Avillez & Breitschwerdt (2005) but none of these studies include the effects of rotation and magnetic fields simultaneously which is a necessary condition for the operation of a galactic dynamo.

Only dynamical simulations can overcome the limitations of the aforementioned analytical models. Statistical properties of the simulated velocity field will allow to determine the regime of the galactic dynamo, i.e. the ratio of flux generation versus flux expulsion as described by the effects (i) and (iii) above. Within the 2nd-order correlation approximation (SOCA, e.g. Rüdiger & Hollerbach 2004) these two effects are characterized by the kinetic helicity and the gradient in the velocity dispersion, respectively. Both quantities are directly accessible from the simulations. Preliminary results already indicate, that the escape velocity V_{esc} has been overestimated in the analytical studies. Only with highly resolved direct numerical simulations it is possible to obtain separation of scales, which is crucial to apply classical mean-field theories and obtain reliable results.

Furthermore, from the described direct numerical simulations we aim to extract (suitably averaged) vertical profiles of the mean magnetic field and turbulent electromotive force. By correlating the self-consistently excited magnetic field with the mean EMF one may derive profiles of the components of the alpha-tensor and thus put constraints on global mean-field models. These models have a long tradition in our working group (Meinel, Elstner & Rüdiger 1990, Schultz, Elstner & Rüdiger 1993) and trace the long-time evolution of the magnetic field while neglecting the small-scale physics. However, the mean-field description is based on the knowledge of the alpha-coefficients. So far the amplitude and functional dependence of the coefficients has rather been chosen for reasons of practicability or even crude estimates have been used. Determining self-consistent values for the alpha-effect would be a huge leap ahead in our understanding of the galactic magnetic field.

References:

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Model description

1 Physical model and equations

We simulate the dynamic evolution of the stratified, turbulent ISM utilizing a 3D MHD model including various physical effects. The computational domain covers a box of $0.5 \times 0.5 \times 4.0 \text{ kpc}^3$ representing a local patch of the galactic disk. Parameters like midplane density, rotation rate Ω and shear parameter q can be adjusted to suit conditions at varying distance from the galactic centre. The implemented equations (in standard notation) read:

$$\begin{aligned}
 \partial_t \varrho + \nabla \cdot (\varrho \mathbf{v}) &= 0 \\
 \partial_t e + \nabla \cdot [(e + p^*) \mathbf{v} - 1/\mu (\mathbf{v} \cdot \mathbf{B}) \mathbf{B}] &= +2\varrho \Omega^2 q x \mathbf{x} \cdot \mathbf{v} - \varrho g \mathbf{z} \cdot \mathbf{v} + \Gamma(\mathbf{r}, t) \\
 &\quad + \nabla \cdot [\mathbf{v} \tau + \eta/\mu \mathbf{B} \times (\nabla \times \mathbf{B}) + \kappa \nabla T] \\
 \partial_t (\varrho \mathbf{v}) + \nabla \cdot [\varrho \mathbf{v} \mathbf{v} + p^* I - 1/\mu \mathbf{B} \mathbf{B}] &= -2\varrho \Omega \mathbf{z} \times \mathbf{v} + 2\varrho \Omega^2 q x \mathbf{x} - \varrho g \mathbf{z} + \nabla \cdot \tau \\
 \partial_t \mathbf{B} - \nabla \times (\mathbf{v} \times \mathbf{B} - \eta \nabla \times \mathbf{B}) &= 0 \\
 \nabla \cdot \mathbf{B} &= 0 \\
 p^* &= p + \mathbf{B}^2/2\mu, \quad p = (\gamma - 1)(e - \varrho \mathbf{v}^2/2 - \mathbf{B}^2/2\mu) \\
 q &= \frac{d \ln \Omega}{d \ln R}, \quad \tau = \nu (\nabla \mathbf{v} + (\nabla \mathbf{v})^\top - 2/3 (\nabla \cdot \mathbf{v}) I)
 \end{aligned}$$

The model includes viscosity ν , magnetic diffusivity η , and thermal conduction κ . The adiabatic equation of

state is supplemented by a parametrized heating- and cooling-function $\Gamma(\mathbf{r}, t)$ allowing for thermal instability. Turbulence is driven by supernova explosions which are modelled as local injections of thermal energy. SN-rates are adopted for typical cited values (cf. Korpi et al. 1999). The model also includes a differentially rotating background and vertical stratification. The shearing boundary conditions in radial direction are implemented to comply with the conservative scheme, for details see Gressel & Ziegler (2007). The initial density and pressure profiles are numerically integrated to be in hydrostatic equilibrium with respect to the equation of state given by the radiative equilibrium. Including z-dependent heating rates this leads to a considerable deviation from usual isothermal initial models. While isothermal models collapse initially this is not the case for our improved stratification.

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2 Numerical methods and algorithms

Version 3 of the NIRVANA code is a general purpose MHD fluid tool capable of the simulation of multi-scale, self-gravitating magnetohydrodynamics problems in three space dimensions employing the technique of adaptive mesh refinement (AMR). The main building blocks of NIRVANA are: (i) a fully conservative, divergence-free Godunov-type central scheme for the solution of the equations of magnetohydrodynamics, Ziegler (2004); (ii) a block-structured mesh refinement algorithm which automatically adds and removes elementary grid blocks whenever necessary to achieve adequate resolution and, (iii) an adaptive mesh Poisson solver based on multigrid philosophy which incorporates the so-called elliptic matching condition to keep the gradient of the gravitational potential continuous at fine/coarse mesh interfaces, Ziegler (2005).

The numerical scheme is based on the Godunov-type central scheme for 2D-conservation laws developed by Kurganov et al. (2001). The scheme has been extended to 3D-MHD and combined with constrained transport (CT) to solve the induction equation (Evans & Hawley 1988) ensuring a divergence-free evolution of the magnetic field. We apply a semi-discrete approach where the spatially discretized equations in flux conservation form are integrated in time via a third-order, TVD Runge-Kutta scheme. Due to the complexity of shearing boundary conditions, these have only been implemented for the non-AMR case and for block wise domain decomposition along the z-direction. However, this does not put a serious constraint on MPI-performance as can be seen from comparing Table 1 with Table 2 (also cf. Caunt & Korpi 2001). The update due to heating and cooling processes is implemented implicitly using a Patankar-type discretization of the corresponding source term which is solved via Newton-Raphson iteration. The CFL time-step is limited such that the relative change in thermal energy due to heating and cooling does not exceed 20% within one time-step. This dramatically increases computational cost but is necessary to properly model the injection of energy.

3 Parallelization of the code

NIRVANA is written in C (~20.000 lines) and has been efficiently parallelized for distributed-memory architectures using asynchronous communication provided by the MPI message-passing library (Ziegler, paper in preparation). Domain decomposition can either be done block wise (non-AMR) or with a space filling curve mapping (AMR). In the latter case load balancing is optimized by runtime repartitioning of the AMR-hierarchy among the MPI-tasks ensuring optimal data locality for both the inter-level and intra-level communications.

# tasks	16	32	64	128
wall-clock time	696s	353s	182s	105s
speed-up	1.0000	1.9717	3.8242	6.6286
efficiency	1.0000	0.9858	0.9560	0.8286

Table 1: Performance (no AMR) with MPI. 10 time-steps at constant problem size $128 \times 128 \times 1024$, domain decomposition along z, only.

# tasks	16	32	64	128
wall-clock time	723s	391s	192s	106s
speed-up	1.0000	1.8491	3.7656	6.8208
efficiency	1.0000	0.9246	0.9414	0.8526

Table 2: Performance (no AMR) with MPI. 10 time-steps, at constant problem size $128 \times 128 \times 1024$, domain decomposition along x,y & z.

# tasks	2	4	8	16	32	64	128	256	512
wall-clock time	4837s	2702s	1389s	641s	337s	179s	84s	44s	24s
speed-up	1.000	1.791	3.484	7.552	14.38	27.06	57.42	109.32	199.47
efficiency	1.0000	0.8953	0.8709	0.9440	0.8984	0.8457	0.8971	0.8540	0.7792

Table 3: same as Table 2, but on IBM Regatta 690+ system JUMP (based on four runs per column shown).

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